Magnetoplasma Oscillations under Anomalous Skin Effect Conditions*

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The frequency of magnetoplasma oscillations are calculated under anomalous skin effect conditions, $kl>1$, k being the wave number of the oscillation and l the electron mean free path. In large magnetic fields such that $\omega_c\tau\gg1$ it is found that for $kl\ll\omega_c\tau$ the frequencies are only slightly modified, whereas for $kl\gg\omega_c\tau$ the oscillations are so heavily damped as to be unobservable.

MAGNETOPLASMA oscillations have been observed at 4°K in small parallelopipeds of very pure metals placed in a large magnetic field.[1,2] The oscillation consists of a standing electromagnetic wave circularly polarized in the same sense as the cyclotron rotation of the electrons. The remarkable property of such waves is that in static fields of several thousand oersteds and a wavelength determined by a sample size of a few millimeters the frequencies are very low, of the order of 100 cps. Such excitations were first predicted by Aigrain[3] and termed "helicon" modes.

The dispersion relation in Gaussian units for such waves is[1]

$$\omega=kk_z(cH_0/4\pi ne), \tag{1}$$

where k, k_z are the magnitude and z component of the wave vector, H_0 is the static magnetic field along the z axis, and there are n electrons per unit volume. This result was derived from a free-electron picture in which damping effects due to the finite mean free path of the electrons are neglected; that is, the cyclotron frequency ω_c is assumed much greater than the inverse relaxation time $1/\tau$. In the original experiments of Bowers, Legendy, and Rose[1] on sodium, only the oscillation of lowest wave number, the fundamental of the block of sodium, was considered. More recently Bowers, Rose, and Taylor[4] have studied the higher harmonics and verified the above dispersion relation. Jones and Chambers,[2] in a series of careful measurements on Na, K, In, and Al, have used Eq. (1) to measure the product ne for each metal. The results suggest that the effective electron charge is several percent less than the free-electron value.

The mean free path in Bowers' sodium sample was very long, about $\frac{1}{4}$ mm. Thus, for harmonics of wavelengths appreciably shorter than $\frac{1}{4}$ mm it is necessary to deal with a nonlocal electrical conductivity, as in the anomalous skin effect. The resulting modification to Eq. (1) is derived below.

Appropriate magnetoconductivity tensors have been derived by Rodriguez[5] and Kjeldaas[6] in their work on acoustic absorption in metals. For a static field H_0 along the z axis, which is perpendicular to the metal surface, the relation between current and electric field is

$$j_\pm=\frac{3\sigma_0E_\pm}{4(1-i\omega\tau\mp i\omega_c\tau)}\int_0^\pi\frac{\sin^3\theta d\theta}{1+ib_\pm\cos\theta}, \tag{2}$$

where $b_\pm=kl/(1-i\omega\tau\mp i\omega_c\tau)$, and l is the electron mean free path. Chambers[7] has pointed out that this equation can be derived from the zero-field case by simply replacing ω by $\omega\mp\omega_c$ in the circularly polarized current components $j_\pm$.

Magnetoplasma resonances are studied in high fields so that $\omega_c\tau\gg1$, whereas the resonance frequency is sufficiently low that $\omega\tau\ll1$. Thus $b\cong ia$, where $a=(kl/\omega_c\tau)$. Under anomalous skin effect conditions $kl>1$, there are two cases to consider, namely, $a\ll1$ and $a\gg1$. By expanding the above integral in powers of a and $1/a$, respectively, we derive the conductivity tensors

$$\sigma_{xx}=\sigma_{yy}=0,\quad \sigma_{xy}=-\sigma_{yx}=-(\sigma_0/\omega_c\tau)(1+\tfrac{1}{5}a^2),\quad \text{for } a\ll1; \tag{3}$$

$$\sigma_{xx}=\sigma_{yy}=(3\pi\sigma_0/4\omega_c\tau)(1/a),\quad \sigma_{xy}=-\sigma_{yx}=-(3\sigma_0/\omega_c\tau)(1/a^2),\quad \text{for } a\gg1. \tag{4}$$

Neglecting the displacement current, Maxwell's equations lead to the result

$$\nabla\times\nabla\times\mathbf{E}=-(4\pi/c^2)(\partial\mathbf{j}/\partial t). \tag{5}$$

For a transverse wave propagating along the z axis, the dependence of E_x, E_y on space and time is specified by $\exp[i(kz-\omega t)]$ so that $\nabla\cdot\mathbf{E}=0$, and hence

$$k^2E_x=(4\pi i\omega/c^2)(\sigma_{xx}E_x+\sigma_{xy}E_y),\qquad k^2E_y=(4\pi i\omega/c^2)(\sigma_{yx}E_x+\sigma_{yy}E_y). \tag{6}$$

For $a\ll1$ there are solutions corresponding to undamped circularly polarized waves with frequency

$$\omega=\pm(k^2c^2/4\pi\sigma_{xy})\cong\pm k^2(cH_0/4\pi ne)(1-\tfrac{1}{5}a^2). \tag{7}$$

In the experiments of Bowers *et al.*[1,4] $\omega_c\tau$ was approxi-

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mately 40, so that even in the extreme anomalous region, $kl \gg 1$, we can still have $a \ll 1$, which gives a negligible correction to the frequency. In more recent experiments Bowers[8] has observed the higher harmonics in a thin slab of sodium, and for $kl \sim 8$ no measurable deviation from the linear ω vs k^2 relation was detected. Here $a = (kl/\omega_c\tau) \sim 0.2$, so that Eq. (7) predicts a correction of only 1%, which is less than the experimental error.

When $a \gg 1$ we find from Eq. (6) that

$$\omega = \pm k^2c^2/4\pi(\sigma_{xy} \mp i\sigma_{xx}). \quad (8)$$

If k is real, then ω is complex, and the wave is damped exponentially in time. This would be observed in the free decay of a standing wave when the wave vector is fixed by the sample dimensions. Because, from Eq. (4) $\sigma_{xx} \gg \sigma_{xy}$, the complex frequency is

$$\omega \cong \pm (k^2c^2\sigma_{xy}/4\pi\sigma_{xx}^2)[1 \pm i(\sigma_{xx}/\sigma_{xy})]$$
$$= \pm k^2(cH_0/4\pi ne)(16/3\pi^2)[1 \pm i(\pi a/4)]. \quad (9)$$

Thus for $a \gg 1$ the oscillations will be completely damped out.

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[8] R. Bowers (private communication).

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Mobility of Electrons in Silver Chloride at High Electric Field*

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The mobility of electrons in specially purified AgCl crystals at high electric field has been investigated in order to clarify the scattering mechanism of fast electrons in ionic crystals. A fast-pulse technique was adopted to observe both the transient photoconductivity and the Hall mobility for photoelectrons. The experimental results show that the mobility of fast electrons at low temperatures is a decreasing function of electric field. An energy dissipation mechanism similar to the scattering due to the acoustical mode of lattice vibration is suggested for these fast electrons.

THE theory of the polaron in ionic crystals usually applies to a "slow electron" in thermal equilibrium with the crystal lattice. As the electron gains energy from an electric field, departing from thermal equilibrium, it is expected that the "slow electron" concept will begin to fail and, in addition, certain differences in the scattering mechanism of electrons may take place. The mobility of electrons in AgCl under high electric field has been investigated in order to clarify these points.[1]

The experiment was made possible by using high-purity crystals zone-refined in a chlorine atmosphere. A total of more than 100 zones was passed through the length of the AgCl ingots contained in quartz tubes. It has been found that the Hall mobility of electrons, μ_H, is greater than $10^4 \text{cm}^2/\text{V sec}$ below 40°K in annealed samples cut out of such ingots.[2]

A fast-pulse technique was adopted to observe both the transient photoconductivity and the Hall mobility μ_H for photoelectrons.[3] Short light pulses having a duration of the order of 1 μsec excite electrons out of the full band into the conduction band during the maximum of a longer electric field pulse whose repetition rate was less than one per second. The transient photoresponse was determined by observing pulse height on an oscilloscope due to the drift motion of carriers in an electric field perpendicular to plane parallel electrodes. The Hall mobility of electrons was measured using the electrode geometry of Redfield.[4]

A typical example of observed pulse heights (proportional to induced charge per pulse, Q) versus electric field is shown in Fig. 1 for a pure AgCl crystal at $T = 6.5$°K. Since the "Schubweg" of electrons, $\mu_d E\tau_t$, is short (negligible saturation or collection effect) and positive holes are much less mobile than electrons in AgCl at low temperatures, Q is given by

$$Q = ne\mu_d E\tau_t/D, \quad (1)$$

where n is the number of electrons released by the light pulse in the volume of crystal; e, the electronic charge; μ_d, the drift mobility of electrons; E, the electric field; τ_t, the average life time of an electron before trapping; and D is the thickness of the crystal.

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